



RESEARCH ARTICLE

STRESS AND VIBRATION ANALYSES OF THE WIND TURBINE BLADE (A NREL 5MW)

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ABSTRACT

Owing to the fast development in the energy field, the demand is increasing to improve energy efficiency and lifetime of wind turbine. Therefore, it's important to understand deeply the behavior of wind turbine under different load conditions. This research paper provides an approach to study and analyze the stresses and deformations under the steady-state condition. Also, it was investigated the vibration characteristics of the NREL offshore 5-MW blade (HAWT) with a long of (61.5 m) and with rotor diameter (126 m). The 3D model of wind turbine blade was created by using SOLIDWORKS and then exported to ANSYS/Workbench19 in order to achieve the numerical simulation based on Finite element method. The steady-state analysis of the selected wind turbine blade was performed at maximum rated power (maximum rotation velocity =12.1 rpm). In this work, three different materials (E-glass fiber, Kevlar, and Carbon fiber reinforced plastic) were selected to build the body of the wind blade parts. The results presented the von-Mises stresses, total deformations, first ten natural frequencies and mode shapes of NREL 5-MW wind turbine blade. In steady-state analysis, it was found that the optimum material was (CFRP) where the minimum level of stresses occurred. In vibration analysis, it was found the material that has a higher structural stiffness is CFRP material which avoids high frequencies and mode shapes.

KEYWORDS

Wind turbine blade, Stress analysis, vibration analysis, finite element method.

1. INTRODUCTION

Renewable energy is energy created from sources that do not deplete, such as solar, wind, hydropower and geothermal. Most renewable energy is consequent from direct or indirect way of exploiting. Renewable energy has two distinctive features, one is infinite reserve, and the second feature is proving clean energy (zero carbon monoxide emission) contrary to other sources of energy such as coal, natural gas, crude oil, and uranium that are harmful to the environment, because they produce toxic gases when burned, one of these gases is carbon dioxide gas causing a change in climate. Climate change has a major impact on the environment such as increased pollution, droughts, rising sea levels and rising temperatures. High temperatures cause changes in environmental integration. So the pressing question today is that, besides providing enough energy for humanity, can it be able to ensure a safe world for the next generation? Thus, there should be a need to learn how to solve the use of alternative energy resources alongside traditional sources. For this reason, renewable and sustainable energy is of interest in the current study [1]. Wind energy is an alternative to fuel, it plays an important role in supplying the most industrialized countries with energy. Wind power production is growing rapidly annually, Figure 1 shows the Global capacity and annual additions of wind power. Wind power is generated by wind turbines, the blades of wind turbine are the most critical component of the wind turbine system. Loads of wind caused a deflection in the wind turbine blades, so the blade should have enough strength with light weight to avoid failure. The blades are characterized by specific strength and high rigidity. Therefore, strong and light materials are required in order to save the turbine from fatigue failure and reduce the overall weight of the wind turbine system [2].

Wind turbine consists of three main parts: tower, nacelle and blades. Tower of wind turbines must be strong and stiff to bear the loads of the

blades, generator, and nacelle, also to bear the fluctuations in wind loads due to blade rotation.

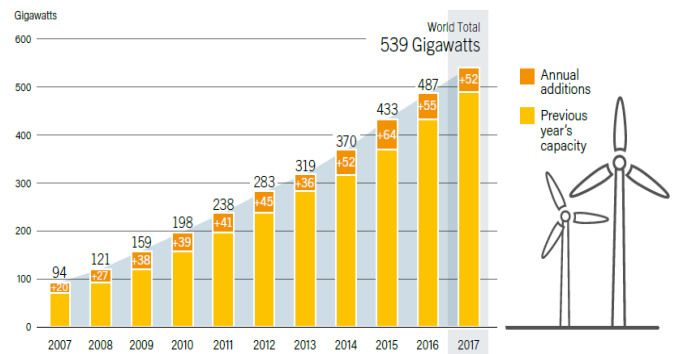


Figure 1: Global capacity and annual additions of wind power, 2007-2017 [3]

Nacelle contains the gearbox, shafts, generator and supporting elements. Blades contain many airfoils with optimum cross sections for aerodynamic efficiency to produce maximum power [3,4]. Many researchers have focused on studying the steady-state and dynamic behavior of different types of wind turbine blades, and calculation the stresses, deformations and natural frequencies of blades by using experimental and numerical approaches.

A study of the influence of rotational speed and thickness on the behavior of the two types of wind turbine blades (NREL 5MW offshore HAWT and WindPACT 1.5MW HAWT) under the steady-state and vibration analyses using four different materials (Structural steel, Aluminum 2024,

Aluminum alloy and Titanium alloy) [5]. The models were built using SOLIDWORKS 2016 software then exported to ANSYS workbench 2016 for analysis. It was calculated the von-Mises stresses, total deformations and first fifth natural frequencies and mode shapes for two models of blades. Krishnamurthy and Sesharao investigated the dynamic characteristics and performance of the blade with NACA 63415 profile [6]. It was focused on the dynamic behavior of a horizontal axis wind turbines blade which exposed to aerodynamic load, centrifugal and the gravity loads. Also, It was studied the effects of these loads on the tip speed ratio and the natural frequency.

In a studied the static and dynamic analysis for small models of blade wind turbine [7]. The blade is modeled by using Pro/Engineer software and analyzed in ANSYS software. Analysis is performed on two different blade materials; E-glass epoxy and S-glass epoxy with twisted angles 15o and 30o. From the results of total deformation, von-misses stress and strain showed that S-glass epoxy material is better than the E-glass epoxy material. Some researchers analyzed the vibration problem of 1.5 MW wind turbine blades by using finite element analysis [8]. It was calculated the natural frequencies and mode shapes. Fatigue analysis was performed by using different materials. It was included the pressure effect on the blade by using the fluent software.

A studied the performance characteristics of the wind turbine blade was studied by changing the blade materials, four different composite materials were used and tested by calculating the total deformation, equivalent (von-Misses) stresses and maximum shear stress [9]. ANSYS Workbench is used for analysis. The static analysis shows the results that Kevlar produces minimum deformations, Epoxy Carbon produces minimum von-Misses stress and Carbon Fiber Reinforced Plastic produces has less maximum shear stresses as compared to other material. Namirian studied the influence of turbulence and gravity loads on the rotor and the tower of a 5MW wind turbine to find the stresses and deflections in the rotor and tower of wind turbine [10]. It was used ANSYS software to design and simulate a full 3-D model of a 5MW wind turbines. The results showed that the increasing in fatigue loads on the structural parts occurred due to the turbulence effect. It was recommended to pay more attention about the fatigue loads, because it's decrease the lifetime of wind turbine components and it causes failure to the turbine components.

The analysis of the stress and deformation of the wind turbine blade 5MW, were also studied using finite element analysis by ANSYS. Wind turbine blade is a sandwich structure made of composite material; the Carbon fiber cloth / epoxy compounds were arranged on the outer layer and fiberglass / vinyl ester as inner layers with PVC foam as the core material [11]. In the analysis, the skin thickness of the blade and stacking angle in the glass fiber/vinyl ester composites were changed. Vibration analysis is an essential element to analyze the dynamic of structure to avoid failure; it will provide the information about the mode shapes and the natural frequencies. It should be noticed that the mode shape and the natural frequency related to the force frequency which applied. In order to obtain an optimum wind turbine design, the vibration should be reduced to minimum level to avoid resonance phenomenon, this phenomenon caused a catastrophic failure in the blade of wind turbine [12].

Steady-state and dynamic behavior were studied for different types of rotating blades. The results of these researchers focused on the stresses and natural frequencies of the damaged and undamaged rotating blades by using numerical and experimental approaches [13-17]. The objective of this research paper is to present an approach to design and analyze the Horizontal-Axis Wind Turbine blade by using finite element method. It was presented an example of wind turbine blade (NREL offshore 5MW blade) to achieve the numerical analysis. The model was built by using SOLIDWORKS19 software and then exported to ANSYS/Workbench19. Two types of analyses were achieved which are steady-state analysis and modal analysis. The result section provides information about the total deformations, von-Mises stresses, first tenth natural frequencies and mode shapes of the selected wind turbine blade. Figure 2 shows the NREL offshore 5MW blade.



Figure 2: NREL 5-MW wind turbine [17]

2. MODELLING OF WIND TURBINE BLADE

Table 1: Dimensions and specifications of the NREL offshore 5-MW HAWT [18]

Rating power (MW)	5 MW
Rotor orientation , configuration	Upwind, 3 blades
Rotor diameter	126 m
Rated rotor speed	12.1 rpm
Rated rotor speed hub height	90 m
Hub Overhung	5 m
Tower base diameter	6 m

In this work, the 3D model of a wind turbine blade was built by using SOLIDWORKS 2018 software. The dimensions and specifications of NREL 5MW wind turbine is shown in Table 1 [18]. The NREL offshore 5MW (HAWT) blade length is 61.5m, where it was divided into 19 sections. The thickness of the outer surface of the blade varies with the length of the blade; the thickness starts at the blade root with 40 mm and reduce sequentially to 20 mm at the blade tip. In order to obtain a successful design a wind turbine blade it should select the appropriate airfoils. Different types of airfoil sections have been used in this design (the DU series and NACA64-618 airfoils) [19]. It was obtained the data for each airfoil based the airfoil tool website. These data can be edited in EXCEL software in order to convert from 2D into 3D, and then inserted into the SOLIDWORK 2018 software.

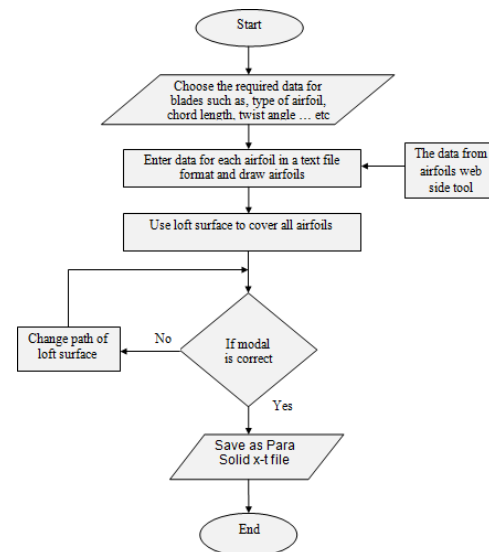


Figure 3: Flowchart of the process to build 3D model of the wind turbine blade

The first step to build the model of the blade is sketched the circular section on the right plane at the origin point and then, drawing the other circular and airfoil sections that selected for each position of blade. In order to obtain the final model, lofting of sections should be successive to obtain 3D blade model [20]. Figure 3 shows the flowchart that explained the processes to build 3-dimensional model of the wind turbine blade. Figure 4 illustrate the airfoil sections of the wind turbine blade. It can be seen the final 3D model of the wind turbine blade [NREL offshore 5-MW blade (HAWT)] in Figure 5.

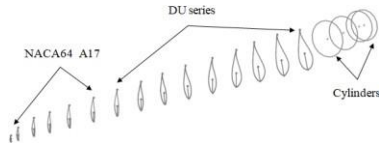


Figure 4: Airfoils sections for the NREL 5-MW blade



Figure 5: NREL offshore 5-MW blade

3. FINITE ELEMENT FORMULATION

This section presents the finite element formulation which used to build the numerical models to find the solutions of the steady-state and vibration problems. The 3D blade model was exported to ANSYS/WORKBENCH19 to analyze the stresses due to the centrifugal effect and free vibration. This work mainly focuses to investigate the behavior of the blade of wind turbine under different working conditions. It was assumed that the blade works at the critical condition, when the output power is the maximum at rated rotational velocity (12.1 rpm).

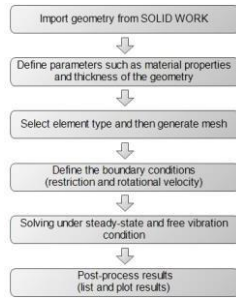


Figure 6: The main steps of finite element analysis

Figure 6 illustrates the flow chart that explain the major steps of finite element method to find the solutions of the steady-state and modal problems. For the simulation, the blade of wind turbine was considered as a cantilever beam fixed at the root end and free at the tip. In this analysis, the effect of rotational speed on the wind turbine blade was studied using three different materials; E-glass fiber, Kevlar, and Carbon fiber reinforced plastic. The mechanical properties of selected materials are shown in Table 2.

Table 2: The mechanical properties of these materials are [19],[20]

Materials	Density (Kg/m ³)	Modulus of elasticity (GPa)	Poisson ratio
E-Glass Fiber	2570	72	0.2
Kevlar	1440	112	0.36
Carbon Fiber Reinforced plastic	1300	13	0.32

Optimal mesh was selected depended on the standard mesh test for both steady-state analysis and modal analysis, the type of mesh used is quadrilateral type for all the part of blade and the number of elements was 10376. Figure 7 shows the selected mesh that used for the steady-stae and modal analyses.

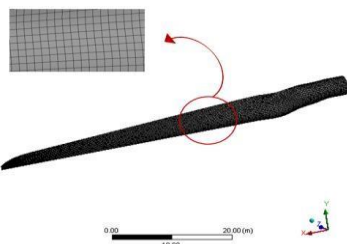


Figure 7: Finite element model of 5-MW turbine blade

It can be written the equation that used to find the response of steady state of wind turbine blades as follows [21]:

$$[K]\{U\}=\{R\} \quad (1)$$

Where $[K]$ is the stiffness matrix of the system and the $\{U\}$ is displacement vectors and $\{R\}$ is load vector (centrifugal force). In the equation (1), it was assumed that the damping forces and inertia forces are equal to zero.

If flexible structure is placed in an appropriate position at $t=0$, the structure can vibrate harmonically. The vibration movement occurs at frequency known as natural frequency and it follows certain patterns known as deformation patterns named mode shapes [22]. The vibration and mode shape of the structure is depends on the stiffness and mass [23]. Assuming that the external force vector $\{R\}$ is zero, then it can be written the equation of the free vibration as follows:

$$[M]\{\ddot{U}\} + [K]\{U\} = 0 \quad (2)$$

And the harmonic displacement is,

$$U_i = \phi_i \sin(\omega_i t + \theta_i) \quad i = 1, 2, \dots, \text{DOF} \quad (3)$$

Where, the symbol ϕ_i is a vector of contractual amplitude for the i^{th} modes of vibration (mode shape), ω_i is the angular frequency of mode i and θ_i is the phases angle. After deriving Equation (3) twice respect to time (t) to obtain the following form,

$$\ddot{U} = -\omega_i^2 \phi_i \sin(\omega_i t + \theta_i) \quad (4)$$

Substitution Eq. (3) and Eq. (4) into Eq. (2), the term $\sin(\omega_i t + \theta_i)$ was deleted to get the following,

$$([K] - \omega_i^2 [M])\phi_i = 0 \quad (5)$$

Eq. (5) is the most effective formula for the structural vibration (Eigen Value Problem) only in the standard model, the Symmetric following:

$$([A] - \lambda_i [I])XX_i = 0 \quad (6)$$

Where, $[A]$ is a dynamic matrix (symmetric matrix), the symbol indicates the i^{th} eigenvalue value, $[I]$ is an identity matrix and XX_i represents the eigenvector corresponding to the new system of this homogeneous equation. Make Eq. (5) as a formula of Eq. (6) by inserting matrix $[K]$ or matrix $[M]$, by using the (Cholesky square root method). It's a straightforward way to solve the linear systems that rely on the fact that any matrix used is a square matrixes $[A]$ can be mentioned as products of the upper and lower triangular matrixes [24].

4. RESULTS AND DISCUSSIONS

In this section the results of steady-state and vibration analyses are presented. The results displayed the stresses, deformations, natural frequencies and mode shapes of the wind turbine blade. The maximum allowable rotational velocity of this type of wind turbine is 12.1 rpm to produce the maximum rated power. In this analysis, the effect of the speed of rotation on the behavior of blade was studied using three different materials, E-glass fiber, Kevlar, and carbon fiber reinforced plastic. Figures 8-13 show the total deformations and Von-Mises stresses of the NREL 5MW wind turbines blade using the selected materials.

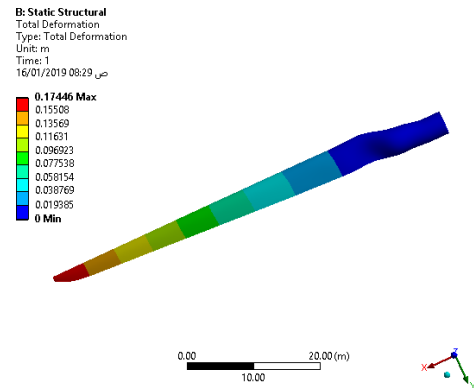


Figure 8: Total deformation of E-glass fiber, rated speed

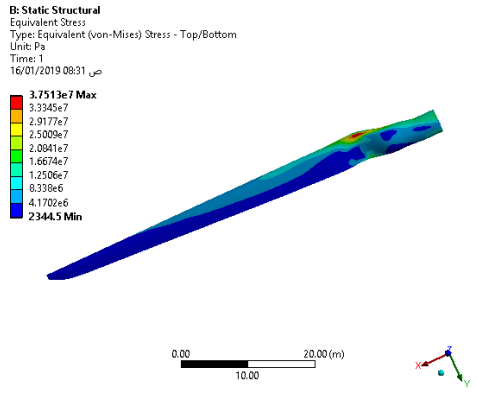


Figure 9: Von-Mises stress of E-glass fiber, rated speed

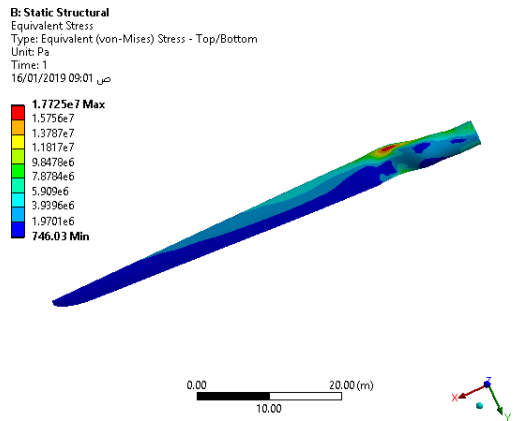


Figure 13: Von-Mises stress of CFRP, rated speed

It can be noticed that the distribution of deformations and stresses are similar when using the three materials (the maximum deformation occurred at the tip region of the blade because it is free to move and the maximum stress occurred in the root region). But the values of stresses and deformations vary with materials; the minimum value of deformation was appeared when used the Kevlar material. It can be seen that the minimum stresses occurred when used the CFRP material.

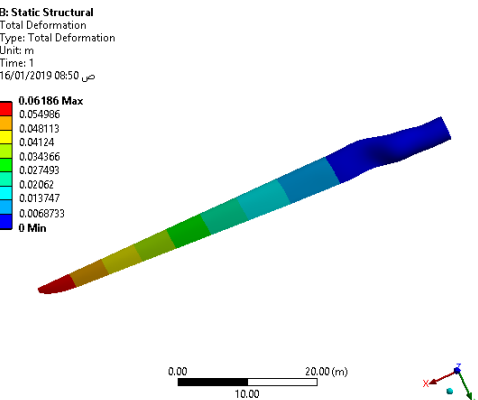


Figure 10: Total deformation of Kevlar, rated speed

The reason for this difference is the weight of blade, where increasing the weight of the blade leads to an increase in the centrifugal force according to the equation of the centrifugal force ($F=m\omega^2r$). Tables 3-5 show the deformations and stresses of the three materials for the different angular velocity in which the turbine rotates. Table 6 lists the first ten natural frequencies of the blade when using the selected materials. Figures 14-22 exhibit the mode shapes of the blade of wind turbine using the selected materials.

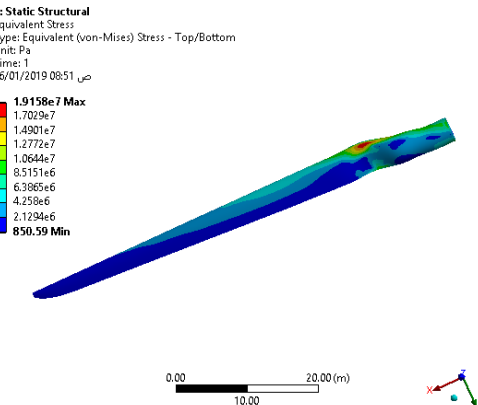


Figure 11: Von-Mises stress of Kevlar, rated speed

Table 3: Total deformation and Von-Mises stresses for E-glass fiber at different rotational speed

Rotational velocity (rpm)	Total deformation (m)	Von-Mises stress (Mpa)
3	0.0105	2.241
6	0.0423	8.965
9	0.0950	20.173
12.1	0.1744	37.513

Table 4: Total deformation and Von-Mises stresses at different rotational speed (kevlar material)

Rotational velocity (rpm)	Total deformation (m)	Von-Mises stress (Mpa)
3	0.003	1.144
6	0.015	4.578
9	0.033	10.303
12.1	0.061	19.158

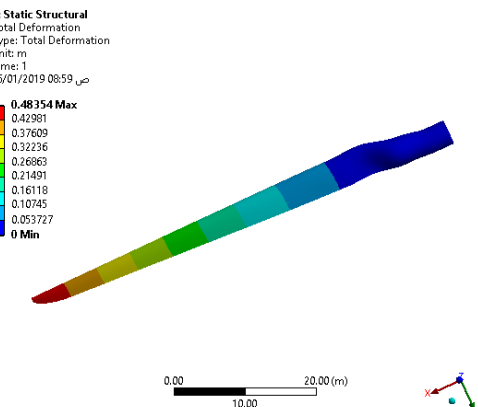


Figure 12: Total deformation of CFRP, rated speed

Table 5: Total deformation and Von-Mises stresses for CFRP at different rotational speed

Rotational velocity (rpm)	Total deformation (m)	Von-Mises stress (Mpa)
3	0.029	1.059
6	0.112	4.236
9	0.263	9.532
12.1	0.476	17.772

Table 6: Natural frequency of NREL 5-MW blade at different materials

Mode shapes	Natural frequency (Hz)		
	E-glass fiber	Kevlar	CFRP
1 st	0.508	0.856	0.306
2 nd	1.417	2.368	0.848
3 rd	1.809	3.028	1.084

4th	4.249	7.093	2.541
5th	5.783	9.645	3.456
6th	7.552	12.634	4.523
7th	8.845	15.029	5.355
8th	10.541	17.814	6.358
9th	10.872	18.445	6.573
10th	12.361	20.84	7.449

C: Modal E glass
Total Deformation 7
Type: Total Deformation
Frequency: 8.8454 Hz
Unit: m
16/01/2019 08:41 ص

0.014982 Max
0.013317
0.011653
0.009988
0.008233
0.006587
0.004994
0.003293
0.001647
0 Min

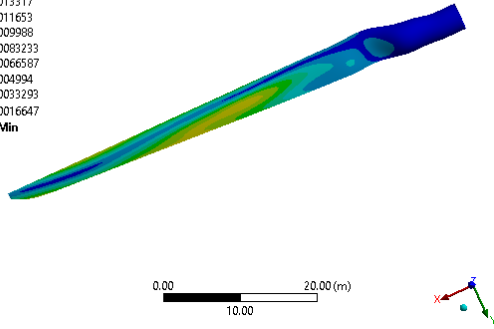


Figure 14: 1st mode shape of E-glass fiber blade

C: Modal E glass
Total Deformation 8
Type: Total Deformation
Frequency: 10.54 Hz
Unit: m
16/01/2019 08:42 ص

0.016182 Max
0.014384
0.012586
0.010788
0.008989
0.0071919
0.005394
0.003596
0.001798
0 Min

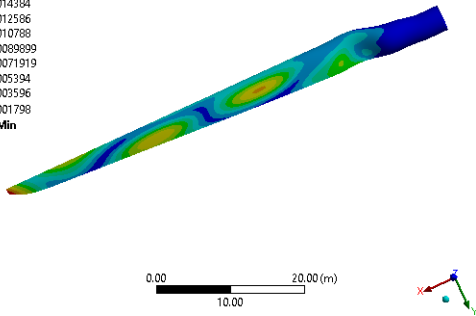


Figure 15: 2nd mode shape of E-glass fiber blade

C: Modal E glass
Total Deformation 9
Type: Total Deformation
Frequency: 10.876 Hz
Unit: m
16/01/2019 08:43 ص

0.020615 Max
0.018325
0.016034
0.013744
0.011453
0.0091624
0.0068718
0.0045812
0.0022906
0 Min

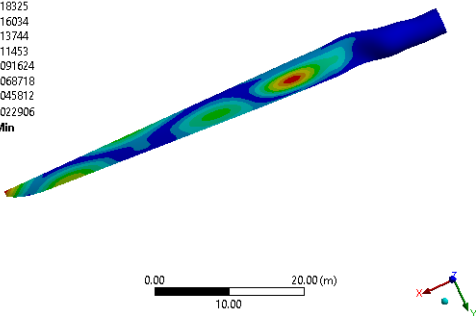


Figure 16: 3rd mode shape of E-glass fiber blade

C: Modal E glass
Total Deformation 7
Type: Total Deformation
Frequency: 15.029 Hz
Unit: m
16/01/2019 08:53 ص

0.018228 Max
0.016203
0.014177
0.012152
0.010127
0.0081014
0.0060761
0.0040507
0.0020254
0 Min

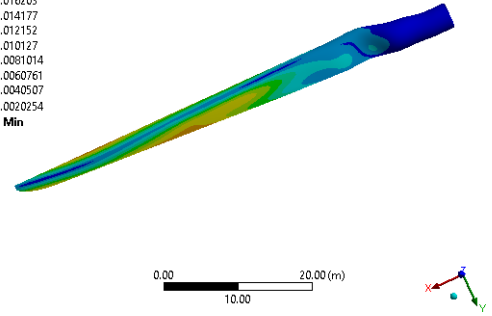


Figure 17: 1st mode shape of kevlar blade

C: Modal E glass
Total Deformation 8
Type: Total Deformation
Frequency: 17.814 Hz
Unit: m
16/01/2019 08:53 ص

0.025117 Max
0.022327
0.019536
0.016745
0.013954
0.011163
0.0083724
0.0055816
0.0027908
0 Min

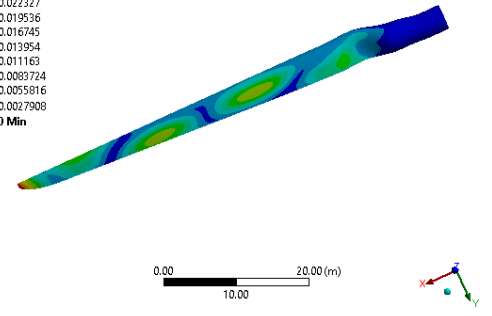


Figure 18: 2nd mode shape of Kevlar blade

C: Modal E glass
Total Deformation 9
Type: Total Deformation
Frequency: 18.445 Hz
Unit: m
16/01/2019 08:54 ص

0.026727 Max
0.023757
0.020788
0.017818
0.014848
0.011879
0.008909
0.0059393
0.0029687
0 Min

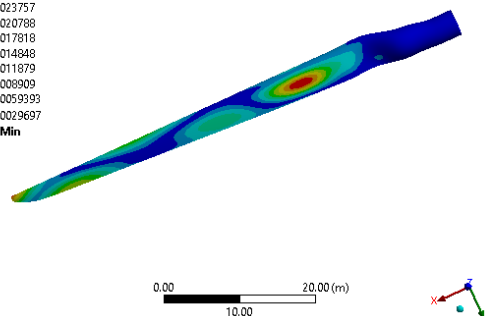


Figure 19: 3rd mode shape of Kevlar blade

C: Modal E glass
Total Deformation 7
Type: Total Deformation
Frequency: 5.3555 Hz
Unit: m
16/01/2019 09:02 ص

0.019759 Max
0.017563
0.015368
0.013172
0.010977
0.0087816
0.0065862
0.0043908
0.0021954
0 Min

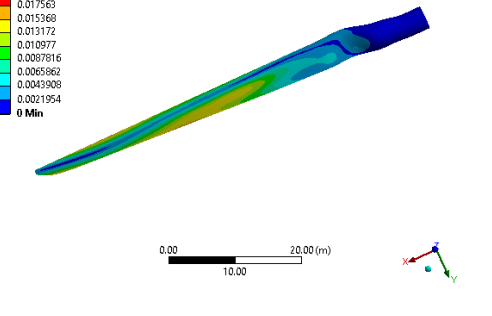


Figure 20: 1st mode shape of CFRP blade

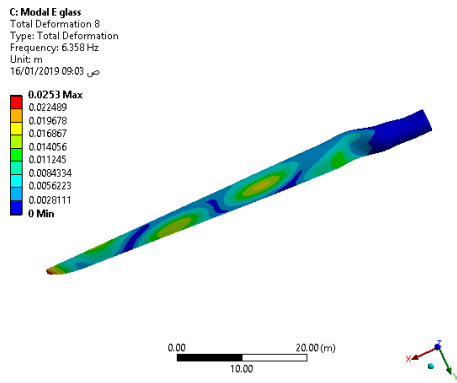


Figure 21: 2nd mode shape of CFRP blade

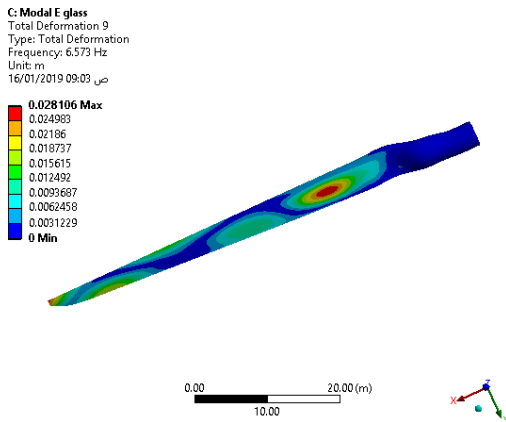


Figure 22: 3rd mode shape of CFRP blade

5. CONCLUSIONS AND REMARKS

Finite element analysis was used to study the behavior of NREL 5-MW wind turbine blade under the steady-state condition. In addition, the vibration characteristics of the blade were investigated deeply. 3-dimensional model of blade of wind turbine have been created using SOLIDWORKS software and then exported to ANSYS/Workbench19 software in order to simulate the steady-state and vibration problems. It was determined the von-Mises stresses, total deformations, natural frequencies and mode shapes of the wind blade. It was found that the stresses decreases when used the carbon fiber reinforced plastic material instead of the other selected materials, the reason of this CFRP has the highest strength. The results of the CFRP are considered satisfactory but it's very expensive material Also, Kevlar is a good material where the level of is acceptable and total deformations were low with compared to other materials.

The stiffness of the structure depends on the properties of the material ρ and E . Therefore, Natural frequencies of the structure depend on the ratio $(\sqrt{E/\rho})$, CFRP material has less natural frequencies than other materials because it has a lower $(\sqrt{E/\rho})$ ratio. Where the ratio $(\sqrt{E/\rho})$ between Kevlar material and E-Glass fiber material is approximately (1.67) which is the same ratio between the ratio $(\sqrt{E/\rho})$ of E-Glass fiber material and CFRP material. It is the same ratio between each natural frequency in a specific mode shape of the blade for the three materials. Basically, this ratio depends on the cantilever beam formula to find natural frequency.

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